Substitution in Consumption and Production; Empirical Testing of the Monocentric City Model

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Lecture 12

Empirical Testing of Monocentric City Model

- Testable propostion: An implication arising from a model which is *falsifiable*, i.e., it is conceivable that empirical* data could be gathered that show the proposition is false.
  - Pellagra is an infectious disease†
  - Light bends under a gravitational pull
  - Other factors held constant an increase in taxes will increase Real GDP
  - 2+2=4
  - 2+2=5

* "originating in or based on observation or experience"

† see Joseph Goldberger at http://www.biography.com/search/article.do?id=9314416

Empirical Testing of Monocentric City Model

- Intensity of Land Use
  - Population density = people/land area
    - Declining density as function of distance from city cents (Fig. 8.1)
    - This follows the equation

$$D(x) = D(x_c)e^{-g(x-x_c)}$$

    - Typical values:
      - X_c=1; pop'n @ x_c =8,000; g=0.4; e≈2.718

Empirical Testing of Monocentric City Model

- Logarithmic transformation:

$$\ln D(x) = \ln D(x_c) - g(x - x_c)$$
$$\ln D(x) = [\ln D(x_c) - gx_c] - gx$$

- Second equation is "linear in the logarithms"; so it can be estimated by OLS; estimates (and t-statistics) appear in Table 8.1, p. 123

Empirical Testing of Monocentric City Model

Statistics of estimation:

- Coefficient estimates
- t-statistics
- R^2
- Adjusted R^2

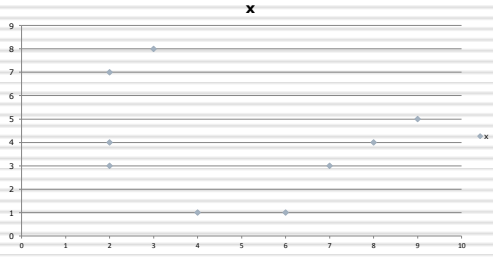
Regression Example: One Explanatory Variable

- Estimate equation: $y=a+bx$
- Means: estimate values of *a* (intercept or constant) and *b* (slope)
- Estimate will minimize sum of squared deviations – called "ordinary least squares" or OLS

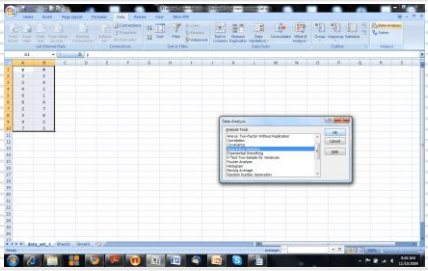
Regression Example: One Explanatory Variable (Data Set 1)

y	x
2	3
2	4
4	1
6	1
8	4
2	7
3	8
9	5
7	3

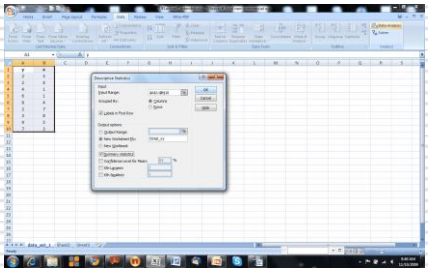
Scatter Diagram of Data



Using Excel for Statistical Analysis – Descriptive Statistics



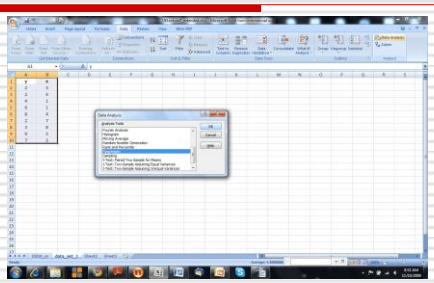
Using Excel for Statistical Analysis – Descriptive Statistics



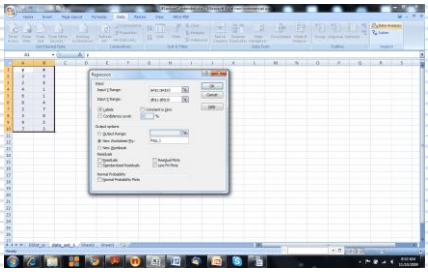
Using Excel for Statistical Analysis – Descriptive Statistics

	y		*x*
Mean	4.777778	Mean	4
Standard Error	0.924629	Standard Error	0.799305
Median	4	Median	4
Mode	2	Mode	3
Standard Deviation	2.773886	Standard Deviation	2.397916
Sample Variance	7.694444	Sample Variance	5.75
Kurtosis	-1.6705	Kurtosis	-0.50446
Skewness	0.385047	Skewness	0.419619
Range	7	Range	7
Minimum	2	Minimum	1
Maximum	9	Maximum	8
Sum	43	Sum	36
Count	9	Count	9

Regression Example: One Explanatory Variable



Regression Example: One Explanatory Variable



Regression Example: One Explanatory Variable - Results

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.225511505
R Square	0.050855439
Adjusted R Square	-0.08473664
Standard Error	2.889021603
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	1	3.130434783	3.130434783	0.375062	0.559618
Residual	7	58.42512077	8.346445825		
Total	8	61.55555556			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*ower 95.0%*	*pper 95.0%*
Intercept	5.821256039	1.957164904	2.974330894	0.02068	1.193296	10.44922	1.193296	10.44922
x	-0.26086957	0.425962997	-0.61242307	0.559618	-1.26811	0.746373	-1.26811	0.746373

Regression Example: One Explanatory Variable - Results

The Estimated Equation:

y = 5.8212 - 0.2608*x

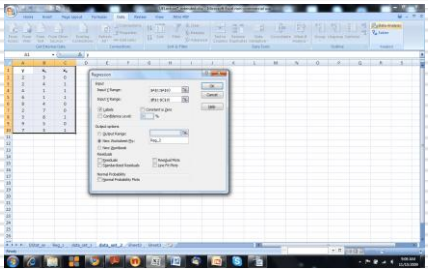
In reality:

y = 5.8212

Regression Example: Two Explanatory Variables – Data Set 2

y	x_1	x_2	
2	3	0	
2	4	1	← Dummy variable
4	1	1	
6	1	1	
8	4	0	
2	7	0	
3	8	1	
9	5	0	
7	3	1	

Regression Example: Two Explanatory Variables - Regression



Regression Example: Two Explanatory Variables - Results

SUMMARY OUTPUT

Regression Statistics

Multiple R	0.328734834
R Square	0.108066591
Adjusted R Square	-0.189244545
Standard Error	3.024991914
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*gnificance F*
Regression	2	6.65209906	3.32605	0.36348	0.709573
Residual	6	54.9034565	9.150576		
Total	8	61.55555556			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*lower 95.0%*	*pper 95.0%*
Intercept	6.89749702	2.684998006	2.568902	0.042398	0.327544	13.46745	0.327544	13.46745
x1	-0.346841478	0.467044471	-0.74263	0.485752	-1.48966	0.795975	-1.48966	0.795975
x2	-1.318235995	2.124924032	-0.62037	0.557827	-6.51774	3.881266	-6.51774	3.881266

Regression Example: Two Explanatory Variables - Results

The estimated equation:

$y=6.8974-0.3468*x_1 -1.318*x_2$

In reality

$y=6.8974$

Regression Example: Three Explanatory Variables - Data

y	x_1	x_2	x_3
2	3	0	1
2	4	1	2
4	1	1	3
6	1	1	4
8	4	0	5
2	7	0	6
3	8	1	7
9	5	0	8
7	3	1	9

Regression Example: Three Explanatory Variables - Results

SUMMARY OUTPUT

Regression Statistics

Multiple R	0.860967202
R Square	0.741247303
Adjusted R Square	0.585995685
Standard Error	1.784806208
Observations	9

ANOVA

	df	SS	MS	F	Significance F
Regression	3	45.62788956	15.2093	4.774490036	0.062607
Residual	5	15.927666	3.185533		
Total	8	61.55555556			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	4.724849095	1.701617542	2.777681	0.03905516	0.350702	9.098996	0.350702	9.098996
x1	-0.844064386	0.31006911	-2.72218	0.041666484	-1.64112	-0.04701	-1.64112	-0.04701
x2	-1.989486922	1.268349436	-1.56856	0.177539206	-5.24988	1.270909	-5.24988	1.270909
x3	0.966891348	0.259267966	3.497852	0.017323456	0.240422	1.573361	0.240422	1.573361

Regression Example: Eliminate Insignificant Variable – Data Set 4

y	x_1	x_3
2	3	1
2	4	2
4	1	3
6	1	4
8	4	5
2	7	6
3	8	7
9	5	8
7	3	9

Regression Example: Eliminate Insignificant Variable – Results

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.783530906
R Square	0.613920681
Adjusted R Square	0.485227575
Standard Error	1.990197936
Observations	9

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	2	37.79023	18.89511	4.770424	0.057548
Residual	6	23.76533	3.960888		
Total	8	61.55556			

	Coefficients	*Standard Err*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	3.285173564	1.597751	2.056123	0.085514	-0.62438	7.19473	-0.62438	7.19473
x1	-0.683549978	0.326379	-2.09434	0.081109	-1.48217	0.11507	-1.48217	0.11507
x3	0.845360825	0.285776	2.958126	0.025344	0.146093	1.544629	0.146093	1.544629

Empirical Testing of Monocentric City Model

Evaluating the Density Gradient

- t-statistics: all above 2
- R^2 "relatively low"; indication of (a kind of) *specification error*
- Alternative functional forms (e.g., polynomials:

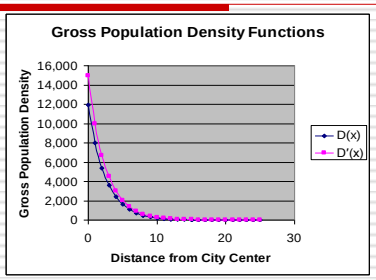
  $lnD(x)=a+g_1x+g_2x^2+g_3x^3$

Empirical Testing of Monocentric City Model

	Base Case	Comparison Case
x_c	1	1
$D(x_c)$	8000	10000
g	0.4	0.4

x	D(x)	D'(x)
0	11,935	14,918
1	8,000	10,000
2	5,363	6,703
3	3,595	4,493
4	2,410	3,012
5	1,615	2,019
6	1,083	1,353
7	726	907
8	486	608
9	326	408
10	219	273

Empirical Testing of Monocentric City Model



Other Measures of Intensity of Land Use

- Floor-area ratio:
  floor space/land area
  regulation of land per unit or land per floor area (see example p. 146)
- Teaser Question: Interpret Table 8.2, p. 132
- Land value gradient
- Housing value gradient?
  What are the problems? p. 136
- Teaser Question: Interpret Table 8.5, p. 136