

Substitution in Consumption and Production; Empirical Testing of the Monocentric City Model

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Lecture 12

Empirical Testing of Monocentric City Model

- Testable proposition: An implication arising from a model which is *falsifiable*, i.e., it is conceivable that empirical* data could be gathered that show the proposition is false.
 - Pellagra is an infectious disease†
 - Light bends under a gravitational pull
 - Other factors held constant an increase in taxes will increase Real GDP
 - $2+2=4$
 - $2+2=5$

* "originating in or based on observation or experience"

† see Joseph Goldberger at <http://www.biography.com/search/article.do?id=9314416>

Empirical Testing of Monocentric City Model

□ Intensity of Land Use

- Population density = people/land area
 - Declining density as function of distance from city cents (Fig. 8.1)
 - This follows the equation

$$D(x) = D(x_c)e^{-g(x-x_c)}$$

- Typical values:
 - $X_c=1$; pop'n @ $x_c = 8,000$; $g=0.4$; $e \approx 2.718$
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Empirical Testing of Monocentric City Model

□ Logarithmic transformation:

$$\ln D(x) = \ln D(x_c) - g(x - x_c)$$

$$\ln D(x) = [\ln D(x_c) - gx_c] - gx$$

- ## □ Second equation is “linear in the logarithms”; so it can be estimated by OLS; estimates (and t-statistics) appear in Table 8.1, p. 123
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Empirical Testing of Monocentric City Model

Statistics of estimation:

- Coefficient estimates
 - t-statistics
 - R^2
 - Adjusted R^2
-

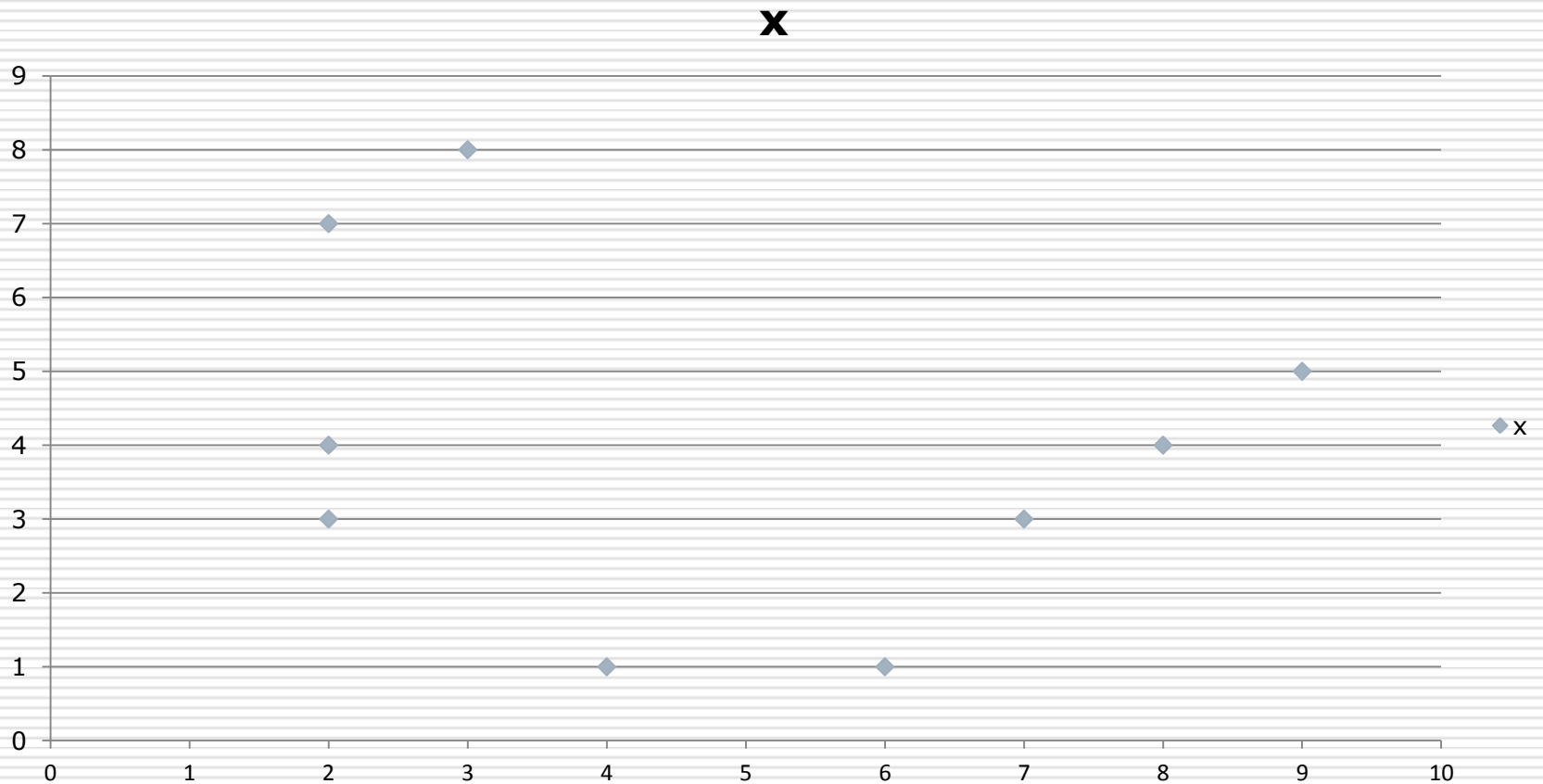
Regression Example: One Explanatory Variable

- Estimate equation: $y = a + bx$
 - Means: estimate values of a (intercept or constant) and b (slope)
 - Estimate will minimize sum of squared deviations – called “ordinary least squares” or OLS
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Regression Example: One Explanatory Variable (Data Set 1)

y	x
2	3
2	4
4	1
6	1
8	4
2	7
3	8
9	5
7	3

Scatter Diagram of Data



Using Excel for Statistical Analysis – Descriptive Statistics

The screenshot displays the Microsoft Excel interface with a data set in the following table:

	A	B
1	y	x
2	2	3
3	2	4
4	4	1
5	6	1
6	8	4
7	2	7
8	3	8
9	9	5
10	7	3

The Data Analysis dialog box is open, showing the following options:

- Anova: Two-Factor Without Replication
- Correlation
- Covariance
- Descriptive Statistics**
- Exponential Smoothing
- F-Test Two-Sample for Variances
- Fourier Analysis
- Histogram
- Moving Average
- Random Number Generation

The status bar at the bottom indicates the current cell is A1, the average is 0, and the zoom level is 100%. The system tray shows the time as 8:44 AM on 11/10/2009.

Using Excel for Statistical Analysis – Descriptive Statistics

The screenshot shows the Microsoft Excel interface with the 'Descriptive Statistics' dialog box open. The data set is as follows:

	A	B
1	y	x
2	2	3
3	2	4
4	4	1
5	6	1
6	8	4
7	2	7
8	3	8
9	9	5
10	7	3

The 'Descriptive Statistics' dialog box is configured with the following settings:

- Input:** Input Range: $\$A\$1:\$B\10
- Grouped By:** Columns, Rows
- Labels in First Row
- Output options:** Output Range, New Worksheet Ply: DStat_xy, New Workbook
- Summary statistics
- Confidence Level for Mean: 95 %
- Kth Largest: 1
- Kth Smallest: 1

The taskbar at the bottom shows the system time as 8:46 AM on 11/10/2009.

Using Excel for Statistical Analysis – Descriptive Statistics

<i>y</i>		<i>x</i>	
Mean	4.777778	Mean	4
Standard Error	0.924629	Standard Error	0.799305
Median	4	Median	4
Mode	2	Mode	3
Standard Deviation	2.773886	Standard Deviation	2.397916
Sample Variance	7.694444	Sample Variance	5.75
Kurtosis	-1.6705	Kurtosis	-0.50446
Skewness	0.385047	Skewness	0.419619
Range	7	Range	7
Minimum	2	Minimum	1
Maximum	9	Maximum	8
Sum	43	Sum	36
Count	9	Count	9

Regression Example: One Explanatory Variable

The screenshot shows a Microsoft Excel spreadsheet with the following data:

	A	B
1	Y	X
2	2	3
3	2	4
4	4	1
5	6	1
6	8	4
7	2	7
8	3	8
9	9	5
10	7	3

The Data Analysis toolpak is open, showing the following options:

- Fourier Analysis
- Histogram
- Moving Average
- Random Number Generation
- Rank and Percentile
- Regression**
- Sampling
- t-Test: Paired Two Sample for Means
- t-Test: Two-Sample Assuming Equal Variances
- t-Test: Two-Sample Assuming Unequal Variances

The status bar at the bottom shows the average of the selected data is 4.3888888.

Regression Example: One Explanatory Variable

The screenshot shows a Microsoft Excel window with the following data in columns A and B:

	A	B
1	Y	X
2	2	3
3	2	4
4	4	1
5	6	1
6	8	4
7	2	7
8	3	8
9	9	5
10	7	3

The Regression dialog box is open, showing the following settings:

- Input Y Range: $\$A\$1:\$A\10
- Input X Range: $\$B\$1:\$B\10
- Labels
- Constant is Zero
- Confidence Level: 95 %
- Output options:
 - Output Range:
 - New Worksheet Ply: Reg_1
 - New Workbook
- Residuals:
 - Residuals
 - Standardized Residuals
 - Residual Plots
 - Line Fit Plots
- Normal Probability:
 - Normal Probability Plots

The taskbar at the bottom shows the system clock at 8:52 AM on 11/10/2009.

Regression Example: One Explanatory Variable - Results

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.225511505							
R Square	0.050855439							
Adjusted R Square	-0.08473664							
Standard Error	2.889021603							
Observations	9							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	1	3.130434783	3.130434783	0.375062	0.559618			
Residual	7	58.42512077	8.346445825					
Total	8	61.55555556						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	5.821256039	1.957164904	2.974330894	0.02068	1.193296	10.44922	1.193296	10.44922
x	-0.26086957	0.425962997	-0.61242307	0.559618	-1.26811	0.746373	-1.26811	0.746373

Regression Example: One Explanatory Variable - Results

The Estimated Equation:

$$y = 5.8212 - 0.2608 * x$$

In reality:

$$y = 5.8212$$

Regression Example: Two Explanatory Variables – Data Set 2

y	x_1	x_2
2	3	0
2	4	1
4	1	1
6	1	1
8	4	0
2	7	0
3	8	1
9	5	0
7	3	1

← Dummy variable

Regression Example: Two Explanatory Variables - Regression

The screenshot shows the Microsoft Excel interface with the 'Data Analysis' task pane open. The 'Regression' dialog box is displayed, showing the following settings:

- Input Y Range: $\$A\$1:\$A\10
- Input X Range: $\$B\$1:\$C\10
- Labels
- Constant is Zero
- Confidence Level: 95 %
- Output options:
 - Output Range:
 - New Worksheet Ply: Reg_2
 - New Workbook
- Residuals:
 - Residuals
 - Standardized Residuals
 - Residual Plots
 - Line Fit Plots
- Normal Probability:
 - Normal Probability Plots

The data table in the background is as follows:

	A	B	C
1	Y	X ₁	X ₂
2	2	3	0
3	2	4	1
4	4	1	1
5	6	1	1
6	8	4	0
7	2	7	0
8	3	8	1
9	9	5	0
10	7	3	1

Regression Example: Two Explanatory Variables - Results

SUMMARY OUTPUT								
<i>Regression Statistics</i>								
Multiple R	0.328734834							
R Square	0.108066591							
Adjusted R Square	-0.189244545							
Standard Error	3.024991914							
Observations	9							
<i>ANOVA</i>								
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>			
Regression	2	6.65209906	3.32605	0.36348	0.709573			
Residual	6	54.9034565	9.150576					
Total	8	61.55555556						
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>
Intercept	6.89749702	2.684998006	2.568902	0.042398	0.327544	13.46745	0.327544	13.46745
x1	-0.346841478	0.467044471	-0.74263	0.485752	-1.48966	0.795975	-1.48966	0.795975
x2	-1.318235995	2.124924032	-0.62037	0.557827	-6.51774	3.881266	-6.51774	3.881266

Regression Example: Two Explanatory Variables - Results

The estimated equation:

$$y = 6.8974 - 0.3468 * x_1 - 1.318 * x_2$$

In reality

$$y = 6.8974$$

Regression Example: Three Explanatory Variables - Data

y	x_1	x_2	x_3
2	3	0	1
2	4	1	2
4	1	1	3
6	1	1	4
8	4	0	5
2	7	0	6
3	8	1	7
9	5	0	8
7	3	1	9

Regression Example: Three Explanatory Variables - Results

SUMMARY OUTPUT									
<i>Regression Statistics</i>									
Multiple R	0.860957202								
R Square	0.741247303								
Adjusted R Square	0.585995685								
Standard Error	1.784806208								
Observations	9								
<i>ANOVA</i>									
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	3	45.62788956	15.2093	4.774490036	0.062607				
Residual	5	15.927666	3.185533						
Total	8	61.55555556							
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>	
Intercept	4.724849095	1.701617542	2.776681	0.03905516	0.350702	9.098996	0.350702	9.098996	
x1	-0.844064386	0.31006911	-2.72218	0.041666484	-1.64112	-0.04701	-1.64112	-0.04701	
x2	-1.989486922	1.268349435	-1.56856	0.177539206	-5.24988	1.270909	-5.24988	1.270909	
x3	0.906891348	0.259267966	3.497892	0.017323456	0.240422	1.573361	0.240422	1.573361	

Regression Example: Eliminate Insignificant Variable – Data Set 4

y	x_1	x_3
2	3	1
2	4	2
4	1	3
6	1	4
8	4	5
2	7	6
3	8	7
9	5	8
7	3	9

Regression Example: Eliminate Insignificant Variable – Results

SUMMARY OUTPUT									
<i>Regression Statistics</i>									
Multiple R	0.783530906								
R Square	0.613920681								
Adjusted R Square	0.485227575								
Standard Error	1.990197936								
Observations	9								
<i>ANOVA</i>									
	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>				
Regression	2	37.79023	18.89511	4.770424	0.057548				
Residual	6	23.76533	3.960888						
Total	8	61.55556							
	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>	<i>Lower 95.0%</i>	<i>Upper 95.0%</i>	
Intercept	3.285173564	1.597751	2.056123	0.085514	-0.62438	7.19473	-0.62438	7.19473	
x1	-0.683549978	0.326379	-2.09434	0.081109	-1.48217	0.11507	-1.48217	0.11507	
x3	0.845360825	0.285776	2.958126	0.025344	0.146093	1.544629	0.146093	1.544629	

Empirical Testing of Monocentric City Model

Evaluating the Density Gradient

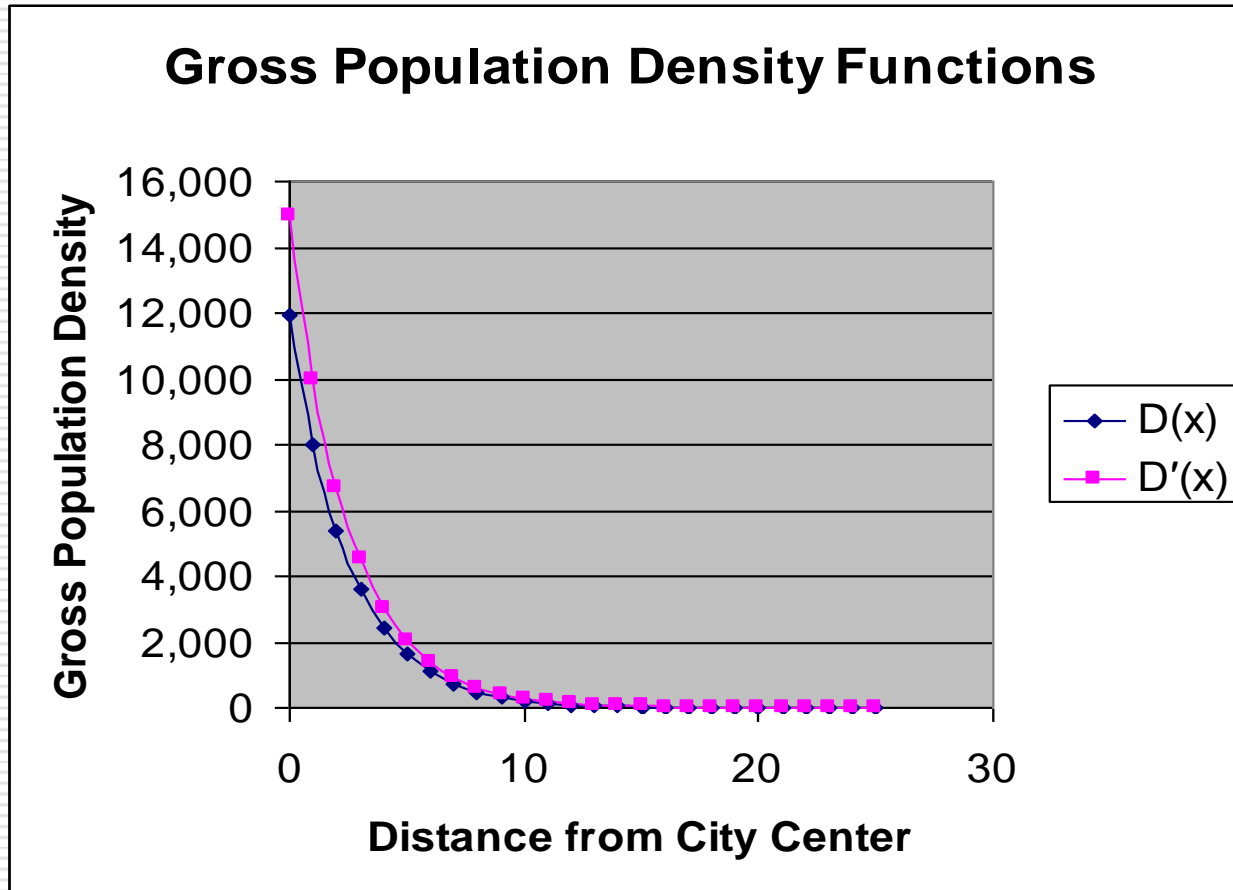
- t-statistics: all above 2
- R^2 “relatively low”; indication of (a kind of) *specification error*
- Alternative functional forms (e.g., polynomials:

$$\ln D(x) = a + g_1 x + g_2 x^2 + g_3 x^3$$

Empirical Testing of Monocentric City Model

	Base Case	Comparison Case
x_c	1	1
$D(x_c)$	8000	10000
g	0.4	0.4
x	$D(x)$	$D'(x)$
0	11,935	14,918
1	8,000	10,000
2	5,363	6,703
3	3,595	4,493
4	2,410	3,012
5	1,615	2,019
6	1,083	1,353
7	726	907
8	486	608
9	326	408
10	219	273

Empirical Testing of Monocentric City Model



Other Measures of Intensity of Land Use

- Floor-area ratio:
floor space/land area
regulation of land per unit or land per floor area (see example p. 146)
 - Teaser Question: Interpret Table 8.2, p. 132
 - Land value gradient
 - Housing value gradient?
What are the problems? p. 136
 - Teaser Question: Interpret Table 8.5, p. 136
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