Methods of Cost-Benefit Analysis

Environmental Waste Management & Urban Infrastructure

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Elements of Cost-Benefit Analysis Easily Stated

- *List* costs
- *List* benefits
- Identify *timing* of costs
- Identify *timing* of benefits
- Determine *discount rate*
- Determine discounted *present value of costs*
- Determine discounted *present value of benefits*
- Subtract DPV of Costs from DPV of Benefits – undertake project if the result is positive, don't otherwise (if all the previous steps have been completed successfully, this step really *is* easy)

Each element is only *apparently* easy

- Difficult to identify all of the *social* benefits and *social* costs
  - Lack of awareness if the costs do not fall on a well-organized and recognized stakeholder
  - Costs and benefits that are difficult to express in monetary terms or whose values are *implicit* rather than *explicit* may be downplayed

Identify *timing* of costs and benefits

- Difficult to predict *project completion* – projects often have delays (may be the result of political jockeying)
- *Useful life* of a project difficult to predict – so the length and size of the stream of net benefits is difficult to predict

Determining the *discount rate*

- The appropriate discount rate is the rate that reflects the social opportunity cost of the resources put into the project
- Different stakeholders are likely to account for different parts of the benefits and costs of the project, so they will apply different social discount rates (innocent explanation)
- Different stakeholders desire different outcomes, so they may lobby for a discount rate favorable to the preferred outcome (political explanation)

Determining the DPV of Costs and Benefits

- The stream of social benefits and social costs may be difficult to determine because some benefits and costs may be difficult to express in monetary terms. They may have *implicit* as opposed to *explicit* values.

Focus of this lecture: *implicit costs*

- Methods for determining implicit costs:
  - We have discussed the *hedonic method* for determining implicit values
  - Now we will discuss another method for determining values for things that don't have explicit prices – *shadow prices*

Context for Discussion of Shadow Prices: *Constrained Optimization*

- Constrained optimization (maximizing or minimizing some objective subject to side conditions) is the most common modeling method in neo-classical economics:
  - Firms maximize profits subject to a variety of constraints, such as the production function
  - Consumers maximize utility subject to the budget constraint
- Common elements in constrained maximization problems
  - Objective function (profits for firms, utility for consumers)
  - Constraints or side conditions (often expressed as *equations* in neo-classical framework)

Linear Programming (LP) Problem

- Elements of the LP Problem
  - *Linear* objective function
  - Linear *inequality* constraints
  - *Non-negativity* conditions
- The *linear* elements in the first two bullet points above is what makes it an LP; if either of these is *non-linear*, then we have a non-linear programming (NLP) problem

Simple Example of an LP Problem

- **Optimal product mix:** Suppose a firm can produce two *outputs*, X and Y, using resources (inputs, such as materials, equipment, labor). Suppose the *resources* are capital (K) and labor (L), and that these resources are available in fixed supply.
- Furthermore, assume that the prices of the outputs (X and Y) are known, but the prices of the inputs (K and L) are unknown.

Unrealistic Example?

- When would we ever not know the prices of the inputs? Actually, there are many cases where the true prices are not known explicitly. This is particularly true at lower levels of a hierarchical organization, where the resource amounts are assigned directly from a higher level of the organization. Examples include …

Optimal Product Mix as an LP Problem

- The *optimal product mix problem* is to choose the amounts of X and Y to produce given the available (and fixed) amounts of K and L, and given the production technologies for producing X and Y, respectively, from K and L
- The optimal product mix problem becomes an LP problem if the *objective function is linear*, the side conditions include *linear inequality constraints*, and if the **choice variables must be non-negative**.

Optimal Product Mix as an LP Problem

- Each of the points in the previous slide corresponds to a specific economic assumption
  - *Linear objective function*: in the optimal product mix the objective function is revenue (price times quantity) for each good produced. Revenue is linear if prices are given or fixed (i.e., if the firm is a price-taker)
  - *Linear inequality constraints*: if the production process is a fixed-proportions production technology (as described below) the resource constraints will be linear inequalities
  - *Non-negativity constraints*: arise naturally in economic problems. E.g., production levels cannot be negative.

Mathematics of the Optimal Product Mix LP Problem

Structure of LP Problem:

Maximize $R = P_xX + P_yY$ ← **objective function**

Subject to:

$a_{Kx}X + a_{Ky}Y \leq K$

$a_{Lx}X + a_{Ly}Y \leq L$

} **linear inequality constraints**

$X \geq 0$

$Y \geq 0$

} **non-negativity constraints**

Mathematics of the Optimal Product Mix LP Problem

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$X \geq 0$

$Y \geq 0$

Interpretation of the Primal LP Problem

- P_x is the price of good X
- P_y is the price of good Y
- P_xX is revenue from selling X units of good X
- P_yY is revenue from selling Y units of good Y
- R is total revenue, which is to be maximized
- a_{Kx} is the number of units of input K required to produce one unit of X; $a_{Kx}X$ is the total amount of K used in producing X units of good X. Likewise $a_{Ky}Y$ is the total amount of K used in producing Y units of good Y.
- These amounts together, $a_{Kx}X + a_{Ky}Y$, must be less than or equal to the fixed amount, K, of capital available.

Specific Example of Optimal Product Mix LP Problem

P_X	3		X	0
P_Y	2		Y	0
a_{Kx}	a_{Ky}	0.5	1	
a_{Lx}	a_{Ly}	3	1	
K	5			
L	12			
objective function		0		

Initial (incorrect) solution

Value of objective function at proposed solution

Specific Example of Optimal Product Mix LP Problem

P_X	3		X	2.8
P_Y	2		Y	3.6
a_{Kx}	a_{Ky}	0.5	1	
a_{Lx}	a_{Ly}	3	1	
K	5			
L	12			
objective function		15.6		

Optimal solution

Value of the objective function at the optimal solution – called the "value of the program"

Dual LP Problem

With every LP Problem is associated another (specific) LP Problem called the Dual LP Problem. (The original LP Problem is called the Primal LP Problem.)

The Dual LP Problem can be constructed in a mechanical way from the Primal LP Problem. That is, there is a strict algorithm for making Dual LP Problems from Primal LP Problems.

Comparing the Primal LP Problem and the Dual LP Problem

Maximize $R = P_x X + P_y Y$	Minimize $C = V_K K + V_L L$
Subject to:	Subject to:
$a_{Kx}X + a_{Ky}Y \leq K$	$a_{Kx}V_K + a_{Lx}V_L \geq P_x$
$a_{Lx}X + a_{Ly}Y \leq L$	$a_{Ky}V_K + a_{Ly}V_L \geq P_y$
$X \geq 0$	$V_K \geq 0$
$Y \geq 0$	$V_L \geq 0$

Making the Dual LP Problem

For the LP Problem given previously, the Dual LP Problem is:

Minimize $C = V_K K + V_L L$

Subject to:

$a_{Kx}V_K + a_{Lx}V_L \geq P_x$

$a_{Ky}V_K + a_{Ly}V_L \geq P_y$

$V_K \geq 0$

$V_L \geq 0$

Making the Dual LP Problem

HOW THE DUAL DIFFERS FROM THE PRIMAL:

Minimize $C = V_K K + V_L L$

instead of maximize

Subject to:

$a_{Kx} V_K + a_{Lx} V_L \geq P_x$

$a_{Ky} V_K + a_{Ly} V_L \geq P_y$

$V_K \geq 0$

$V_L \geq 0$

Making the Dual LP Problem

HOW THE DUAL DIFFERS FROM THE PRIMAL:

Minimize $C = V_K K + V_L L$

Subject to:

$a_{Kx} V_K + a_{Lx} V_L \geq P_x$

$a_{Ky} V_K + a_{Ly} V_L \geq P_y$

$V_K \geq 0$

$V_L \geq 0$

These parameters were in the constraints in the Primal LP Problem; now they are in the objective function of the Dual LP Problem

V_K and V_L are new choice variables that appear in the objective function, the inequality constraints, and the non-negativity constraints

Making the Dual LP Problem

HOW THE DUAL DIFFERS FROM THE PRIMAL:

Minimize $C = V_K K + V_L L$

inequalities reversed

Subject to:

$a_{Kx} V_K + a_{Lx} V_L \geq P_x$

$a_{Ky} V_K + a_{Ly} V_L \geq P_y$

$V_K \geq 0$

$V_L \geq 0$

parameters formerly in the objective function of the Primal LP Problem appear in the constraints of the Dual LP Problem

coefficients a_{ij} *transposed* – rows interchanged with columns

Dual of the Specific Example of Optimal Product Mix LP Problem

P_X	3		V_K	1.2
P_Y	2		V_L	0.8
a_{Kx}	a_{Ky}	0.5	1	
a_{Lx}	a_{Ly}	3	1	
K	5			
L	12			
objective function		15.6		

Optimal solution

Value of the dual program

True and not true statements about the Primal and Dual LP Problems

- The Primal LP Problem and the Dual LP Problem have the same solution.
- **FALSE!!!!**
- The solution to the primal problem is X=2.8 and Y=3.6. The solution to the dual problem is V_K=1.2 and V_L=0.8. **In fact, the primal and dual programs do not even have to have the same number of choice variables.**
- If the primal problem has a solution, so does the dual problem, and value of the primal program is equal to the value of the dual program.
- **TRUE!!!!**
- The value of the primal program is 15.6; so is the value of the dual program.
- The Dual LP Program of the Dual LP Program is the Primal LP Program.
- **TRUE!!!!**

Interpreting the Primal LP Problem

- The Primal LP Problem: To maximize revenue from producing X and Y with the given technology (represented by the a_{ij} coefficients) and the available resources (represented by the fixed amounts of K and L – there are 5 units of K available and 12 units of L) you should produce 2.8 units of X and 3.6 units of Y. Producing this combination at the given prices will yield 15.6 units of revenue. Any other combination of X and Y either produces less revenue or is infeasible.

Interpreting the Dual LP Problem

- How much is capital (input K) worth? How much is labor (input L) worth? We don't know because we were given only **stocks** of these resources. But the solution to the dual LP problem gives us the valuation (or shadow prices) of the resources – V_K=1.2 and V_L=0.8.
- What do these valuations mean? If we had one more unit of capital, we could increase the objective function of the primal problem by 1.2. If we had one more unit of labor, we could increase the value of the objective function by 0.8
- The dual solution variables identify the "bottleneck" resources – any resource not a bottleneck has a zero shadow price, and also gives us a relative value of the bottlenecks.

Applying Linear Programming to Solid Waste Management

Based on Pogodzinski and Ledesma "Waste Not, Want Not"

One of the most significant changes in solid waste management has been the increasing rate of diversion of waste from garbage to recycling and composting. Greater diversion means that the proportion of solid waste that is garbage is falling, while the proportions that consist of recyclables and compostables are rising. This change has important implications for the financing of waste removal, for the geography of waste removal, for investment in waste-handling capacity and infrastructure, and for the level of emissions associated with waste management.

Solid Waste Management

Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Solid waste consists of three streams: garbage (G), recyclables (R), and compostables (C). Each of these waste streams is handled differently. The facilities to handle each stream are typically located in different places, subject to different constraints, have different costs, and the financing of each waste stream is different.

Solid Waste Management
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Increased diversion of waste from garbage to recycling and composting has been a significant goal of public policy. Diversion from landfills would reduce greenhouse gases (see http://www.epa.gov/outreach/sources.html). Additionally, lowering ton-miles hauled would reduce transportation emissions, as would a shift in mode of transportation, say, from trucks to rail.

Landfills are the second most significant source of methane emissions, according to the EPA

Policy Paradoxes and Pricing
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

- There are, however, some paradoxes in increased diversion. Because disposal rates are often set by contract for years and are frequently based on charging only for garbage disposal, increasing diversion may worsen the financial situation of a municipality. Furthermore, considering the entire set of fees and charges associated with disposal of waste, the most cost-effective allocation of waste streams to disposal sites is not necessarily one that optimizes the carbon emissions associated with transportation.
- Both of these paradoxes can be viewed as pricing problems. Basing rates on a "snapshot" of the waste streams at one point in time guarantees a fiscal imbalance if the composition of the waste streams changes. Using an incorrect price of carbon emissions means that there won't be sufficient incentives to economize on waste hauling. Alternative rate-setting practices would reduce the fiscal imbalance. Raising prices of carbon-emitting activities through corrective taxes would lead municipalities and disposal site operators to appropriately account for the effects of carbon emissions in their waste generating, hauling, and disposal practices.

Solid Waste Management
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

As the structure of solid waste changes so does the optimal allocation of waste streams to landfills and other facilities, as well as the capacity limits which constitute the most significant constraints. The change in the structure of solid waste has implications for local public finance. Municipalities typically receive payments based on tonnage deposited at sites within their jurisdiction. Waste facilities may provide substantial revenues to local governments. A municipality can "export" waste disposal services to neighboring cities, and gain revenue and jobs, especially for low- to moderate-skilled workers.

Solid Waste Management
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Alternative objective functions

- **MIN-EXP:** minimize expenditure to dispose of given waste
- **MIN-HAUL:** minimize hauling distance to dispose of given waste
- These are two different Primal LP Programs

Solid Waste Management
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

- The base case in this discussion (corresponding to zero percent diversion) is calibrated, roughly, to the experience of the City of San Jose in 2004. This base case assumes that all shipments originate from a central collection facility. The amount of solid waste disposed in that year was about 6,800 tons per day, with approximately 24% of that total being garbage, 44% recycling, and 32% composting. Our calculations based on the model should be viewed as approximations for illustrative purposes only.
- In the MIN-EXP LP model, expenditures depend on the size of each waste stream, the "tipping fee" (per ton disposal fee) at a particular disposal site, the surcharge of the local jurisdiction (if not the jurisdiction's own – in which case the payment of the surcharge will be returned to the General Fund from which it is paid), and the cost of transporting the waste to the facility.

The "tipping" fee
Tipping fees are a large part of the cost of disposing of waste streams

Solid Waste Management
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

The basic economics of determining the optimal allocation of a waste stream in the MIN-EXP model is to determine the facility with the lowest *total* cost per ton capable of handling that waste stream, and allocate all the waste in that stream to that facility. If capacity is reached at that facility, the remainder of that waste stream is then allocated to the facility with the next lowest total cost per ton until capacity of that facility is reached, and so on, until all the waste in that stream is disposed of. Some waste may go to more distant facilities if the combination of tipping fees and surcharges at the distant facility is low enough so that it more than compensates for the cost of shipping a greater distance. Mode of transportation is decisive in the transportation cost calculation. This is notably the case if a waste facility has rail access, which makes cost of transportation per ton-mile a fraction of the cost of truck transportation.

Optimal Solution to the MIN-EXP Primal Linear Programming Problem in the Base Case
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

G	R	C	
0	0	0	**Altamont Landfill**
1600	0	700	**Newby Island**
0	3000	0	**Ox Mountain Landfill**
0	0	0	**Ostrom Road Landfill**
0	0	0	**Forward Landfill**
0	0	0	**Jepson Prairie Organics**
0	0	0	**Grover Composting Facility**
0	0	1500	**Z-Best Composting**

Solid Waste Management – MIN-EXP
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Total expenditure on waste disposal at this expenditure-minimizing solution is $498,030 and a total of 198,500 ton-miles are hauled. This is *not* the minimum ton-miles to dispose of these waste streams.

Optimal Solution to the MIN-HAUL Primal Linear Programming Problem in the Base Case
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

G	R	C	
0	0	0	**Altamont Landfill**
1600	2000	980	**Newby Island**
0	1000	0	**Ox Mountain Landfill**
0	0	0	**Ostrom Road Landfill**
0	0	0	**Forward Landfill**
0	0	0	**Jepson Prairie Organics**
0	0	0	**Grover Composting Facility**
0	0	1220	**Z-Best Composting**

Total expenditure on waste disposal at this ton-mile minimizing solution is more than it was for the expenditure-minimizing solution, $546,602, but the ton-miles hauled decrease to 130,940. To minimize ton-miles, we dispose of as much waste as possible at the closest facility, Newby Island. The Newby Island facility has reached capacity for recycling and composting at the ton-miles optimal solution, and the remainder of each stream go to the nearest respective facilities that can accommodate that stream.

Composting capacity is the most significant constraint in the San Francisco Bay Area

Shadow Prices for Composting Capacity at Various Facilities at 70% Diversion Rate
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Landfill	Shadow Price	Current Composting Capacity
Altamont Landfill	$0.00	3000
Newby Island	$23.22	980
Ox Mountain Landfill	$50.41	0
Ostrom Road Landfill	$10.97	0
Forward Landfill	$51.22	0
Jepson Prairie Organics	$0.00	750
Grover Composting Facility	$20.50	125
Z-Best Composting	$34.62	1500

Each additional ton of composting capacity at the Forward Landfill would reduce spending on the optimal waste disposal allocation by $51.22. However, currently Forward has no compost waste capacity. This may be because the site is unsuitable technically or politically sensitive. Altamont has a zero value for expansion, because, although some compost is sent there, unused capacity remains at that site.

Optimal solution to MIN-EXP LP problem with growth of solid waste stream and modest diversion to composting
Based on Pogodzinski and Ledesma "Waste Not, Want Not"

Looking into the future, if we assume a growth in tonnage of 175%, diversion of 50%, and also assume that most of the diversion is into recycling and not composting (90% vs. 10%), we arrive at a solution for the MIN-EXP LP problem where almost all landfills are actively used (the exception is Ostrom Road Landfill) and composting is almost at capacity. This solution is illustrated in the map. The red dots indicate actively used facilities. At a higher rate of growth of tonnage or a greater proportion of diversion to composting, the LP problem becomes infeasible!

Demo

LP_simple_example.xls